

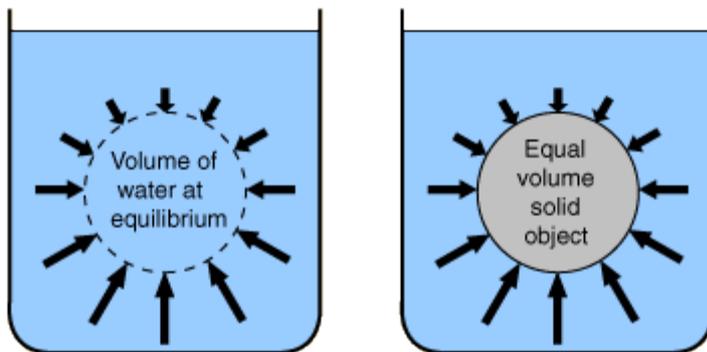
## Chapter 3

### Flotation

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#### Buoyancy

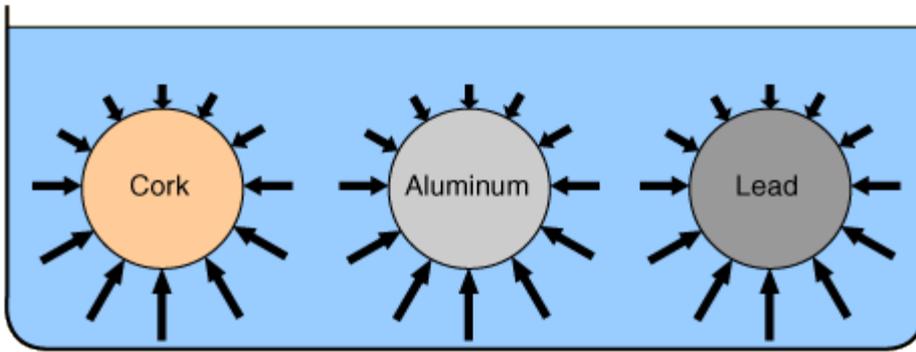
Buoyancy arises from the fact that fluid pressure increases with depth and from the fact that the increased pressure is exerted in all directions (Pascal's principle) so that there is an unbalanced upward force on the bottom of a submerged object.



Since the "water ball" at left is exactly supported by the difference in pressure and the solid object at right experiences exactly the same pressure environment, it follows that the buoyant force on the solid object is equal to the weight of the water displaced (Archimedes' principle).

#### Equal Volumes Feel Equal Buoyant Forces

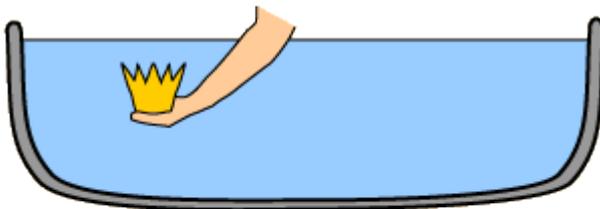
Suppose you had equal sized balls of cork, aluminum and lead, with respective specific gravities of 0.2, 2.7, and 11.3 . If the volume of each is 10 cubic centimeters then their masses are 2, 27, and 113 gm.



Each would displace 10 grams of water, yielding apparent masses of -8 (the cork would accelerate upward), 17 and 103 grams respectively.

The behavior of the three balls would certainly be different upon release from rest in the water. The cork would bob up, the aluminum would sink, and the lead would sink more rapidly. But the buoyant force on each is the same because of identical pressure environments and equal water displacement. The difference in behavior comes from the comparison of that buoyant force with the weight of the object.

### Archimedes' Principle



$$\text{Mass of object} - \text{Apparent mass when submerged} = \text{Density of water} \times \text{Volume of object}$$

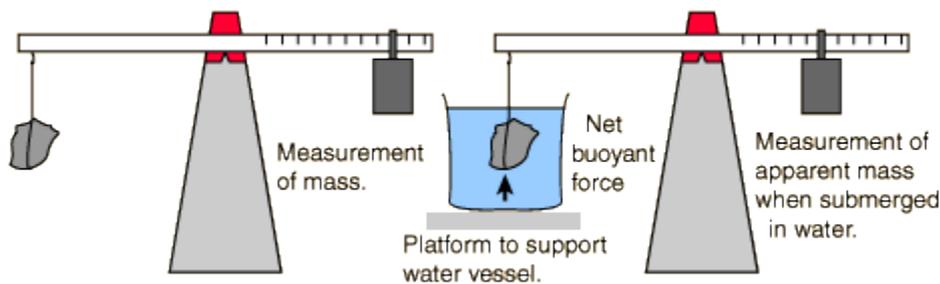
$$440 \text{ grams} - 409 \text{ grams} = 1 \text{ gram/cm}^3 \times \text{volume}$$

$$\text{So the volume must be } 31 \text{ cm}^3!$$

The buoyant force on a submerged object is equal to the weight of the fluid displaced. This principle is useful for determining the volume and therefore the density of an irregularly shaped object by measuring its mass in air and its effective mass when submerged in water (density = 1 gram per cubic centimeter). This effective mass under water will be its actual mass minus the mass of the fluid displaced. The difference between the real and effective mass therefore gives the mass of water displaced and allows the calculation of the volume of the irregularly shaped object (like the king's

crown in the Archimedes story). The mass divided by the volume thus determined gives a measure of the average density of the object. Archimedes found that the density of the king's supposedly gold crown was actually much less than the density of gold -- implying that it was either hollow or filled with a less dense substance.

Examination of the nature of buoyancy shows that the buoyant force on a volume of water and a submerged object of the same volume is the same. Since it exactly supports the volume of water, it follows that the buoyant force on any submerged object is equal to the weight of the water displaced. This is the essence of Archimedes principle.



Archimedes' principle aids in the experimental determination of density by providing a convenient and accurate method for determining the volume of an irregularly shaped object, like a rock.

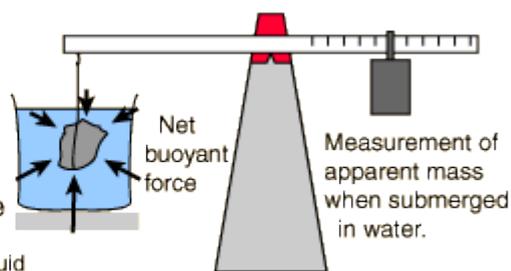
Buoyant force = weight of water displaced.

$$F_{\text{buoyant}} = \rho_{\text{fluid}} gV$$

where  $V$  = volume of submerged object.

But since  $V = \frac{m}{\rho_{\text{object}}}$ , the buoyant force

can be expressed as  $F_{\text{buoyant}} = mg \frac{\rho_{\text{fluid}}}{\rho_{\text{object}}}$

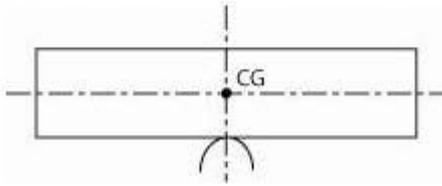


## Center of Gravity

The center of gravity (CG) is the point that represents the average location of all of the weight of an object. The weight of an object is distributed evenly about its center of gravity. As a result, the downward force of an object's entire weight seems to act through its center of gravity.



Because of this, the weight of an object can be balanced by a supporting force, a single upward force applied directly beneath the center of gravity. When you place a wood block horizontally on your finger so that it balances, for example, your finger is directly beneath the block's center of gravity. The weight of the block is distributed evenly about this point.



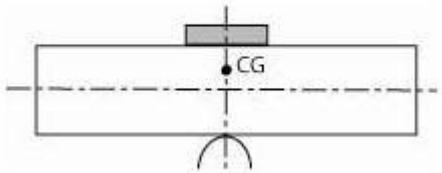
In the diagram (left), imagine the force of the weight acting downward through the CG with the supporting force of the finger pushing upward in the direction of the CG. This upward force exactly matches the weight of the block, even though the weight of the block is distributed around the CG.



These two opposing forces can also be represented by opposing arrows, with the upward supporting force directly below the weight acting through the center of gravity. The two arrows are equal in length, indicating that the strengths of the two forces are exactly the same.



The position of the center of gravity of an object changes if there is a change in the distribution of weight. For example, if we place a coin on top of the wood block, the location of the center of gravity for the combination of the block and the coin is a little higher than for the block alone, because of the added weight of the coin. However, the block and coin can still be supported by a finger pushing upward beneath the center of gravity of the block-and-coin combination.



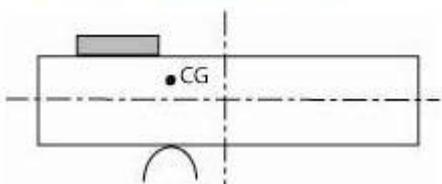
In the diagram (left), imagine the force of the weight acting downward through the CG. The single force of the finger pushing upward in the direction of the CG supports the block and coin.



This can still be represented by two equal but opposing arrows, one for the downward weight of the block and the coin, and the other for the supporting force of the finger pushing upward.

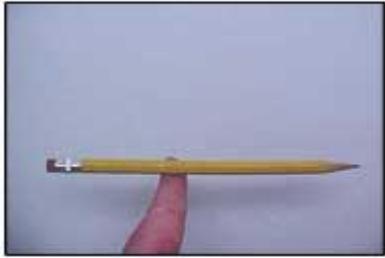


If the coin is moved toward one end of the block, the center of gravity moves toward that end, as well. This is because the weight distribution has changed. However, the block and off-center coin can still be supported by a finger pushing upward beneath the new center of gravity of the block-and-coin combination. Note the slightly off-center location of the supporting finger in the photograph below.



In the diagram (left), imagine the force of the weight acting downward through the CG. The single force of the finger pushing upward in the direction of the CG supports the block and coin. For the off-center coin, the finger pushes upward directly beneath the off-center CG.

Because the weight of the block and the off-center coin acts through the CG, this can still be represented by two equal but opposing arrows, one for the downward weight of the block and the coin and the other for the supporting force of the finger pushing upward.



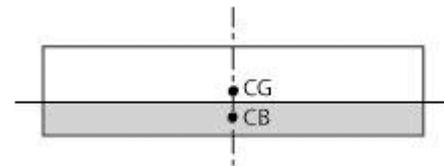
When you balance a pencil, your finger is not at the center of the pencil for the same reason, because of the extra weight of the metal ring.

## Center of Buoyancy

Now it is time to combine these ideas.

The *center of buoyancy* of a floating object is related to the center of gravity concept. When an object is partly or completely submerged in a liquid, some of that liquid is displaced. The volume of displaced liquid is exactly the same as the volume of whatever part of the object is below the surface. The shape of the volume of displaced liquid is also exactly the same as the shape of the part that is submerged, because the object itself is displacing the liquid.

The *center of buoyancy* of the object is located at the center of gravity of the volume of the displaced liquid. It is the point through which the upward buoyant force seems to act.



The diagram here shows a block of wood floating in water. The center of gravity (CG) of the block is at the geometric center of the block, as it was before the block was placed in the water. The center of buoyancy (CB) is at the *center of gravity of the displaced water*.

The challenge is for you to imagine the part of the block below the water level, shaded in gray, being replaced by water. Doing this, however, shows you exactly what the displaced water looks like. In this case, the shape of the displaced water is the shape of the part of the block that is beneath the water surface.

The block floats at a level that displaces exactly enough water to create an upward buoyant force equal to the downward weight of the block.

*Note:* The wood block floats because of Archimedes' principle. It is able to displace enough water to balance its weight without having to submerge its entire volume to do so. Another way of saying this is that the density of the material in the block is less than that of water.



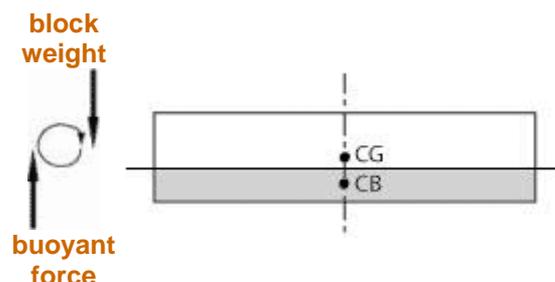
## Stability and Center of Buoyancy

Finally, we can look at stability and buoyancy together.

If the floating wood block is pushed down at one end slightly, the block tilts in the direction of the downward push. This changes the shape of the displaced water and shifts the location of the center of buoyancy.

In the diagram (above right), there is more displaced water on the left side. Therefore, the center of buoyancy is located to the left of center.

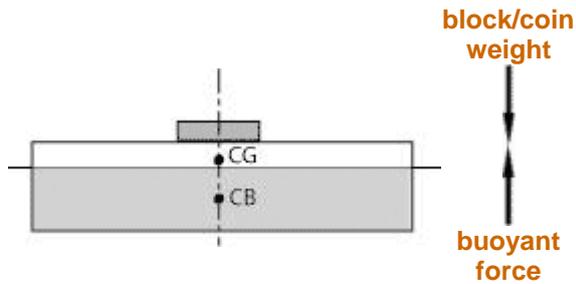
The buoyant force acting upward through the center of buoyancy is the restoring force. When the downward push is removed, the buoyant force and the weight acting through the center of gravity rotate the block back to its original position.



The floating block is *stable*.

If a coin is placed on top and in the middle of a wood block floating in water, the block sinks into the water until an upward buoyant force is created that is equal to the weight of the block and the coin together. Then it stops sinking, because the upward buoyant force exactly balances the downward

combined weight of the block and coin.



Notice in the diagram here that the volume of the displaced water has increased, since the block is riding lower in the water than it was without the coin. The location of the center of buoyancy is at the geometric center of this volume of displaced water.

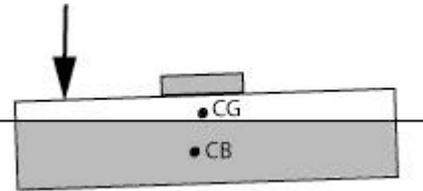
The weight of the block and the coin together acts downward through the center of gravity. The corresponding buoyant force acts upward through the center of buoyancy.

If the block-with-coin is pushed down at one end slightly, it tilts in the direction of the downward push just as it did without the coin.

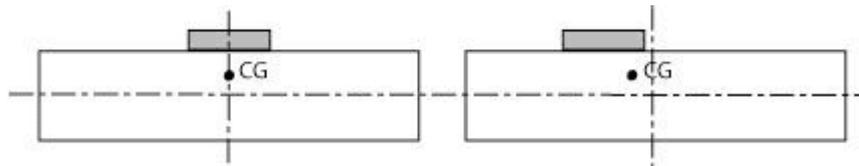


Once again, the center of buoyancy shifts to the left of center. The buoyant force acting upward through the center of buoyancy is the restoring force. When the downward push is removed, the restoring force tilts the block back to its original position.

The floating block with the coin on top in the middle is stable because it returned to its equilibrium position.



If the coin is moved off center toward one end of the block, the center of gravity of the block and coin together moves off center toward the coin.



Because the center of gravity is now off center, the floating block-with-coin tilts downward on the side with the coin. This changes the shape of the displaced water and shifts the location of the center of buoyancy.

This does not change the total volume of displaced water, however, because the weight of the block and coin together is still the same.

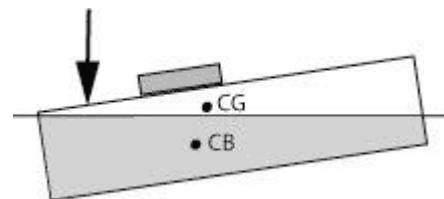
In the diagram (far right), there is more displaced water on the left side. Therefore, the center of buoyancy is located to the left of center. The block continues to tilt until the center of buoyancy is directly beneath the center of gravity.



At this point, the upward buoyant force acting at the center of buoyancy exactly balances the downward force of the combined block-and-coin weight acting through the center of gravity.

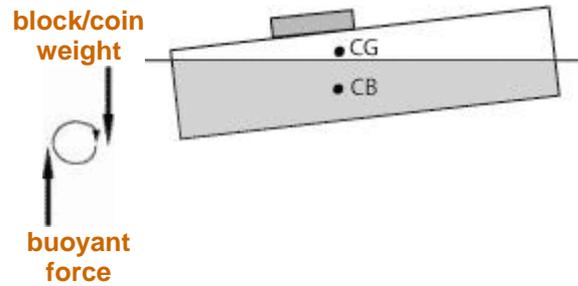
*Note:* The coin stays on the block because the equilibrium angle is not steep enough to cause it to slide downward. If the coin were placed nearer the end, the floating block might tilt enough for the coin to slide into the water.

Pushing down on the side of the block near the coin will cause the block to tilt downward even more. As a result, the shape of the displaced water changes, shifting the location of the center of buoyancy to the left of the center of gravity. This is shown in the diagram below.



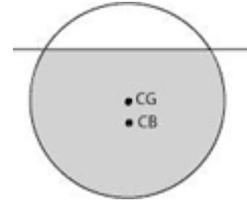
This creates a buoyant force that acts upward at the center of buoyancy as a restoring force. When the downward push is removed, the restoring force tilts the block back to its original tilted position.

The block with the off-center coin is stable.

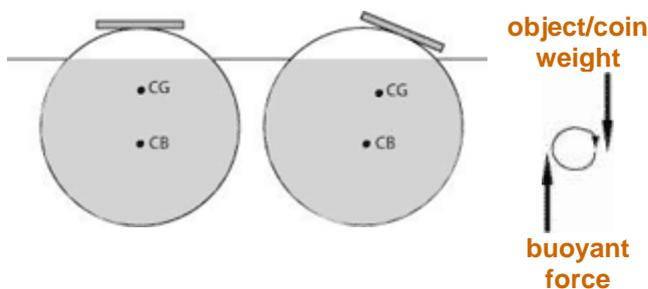


A floating object that is round does not change the shape of its displaced water when it rotates or tilts. Therefore, it cannot use its buoyant force as a restoring force.

If the floating object in the diagram above rotates, the center of gravity and the center of buoyancy stay in the same relative location.



However, if a coin is placed on top of a round floating object, the situation is unstable. In actuality, it is very difficult to place a coin exactly over the center of gravity. The smallest offset is all that is necessary to start the object and the coin to rotate. As it does so, the center of gravity of the object and coin together moves outside of the center of buoyancy.



In this case, the buoyant force and the weight act together to continue the rotation. This continues until the coin drops off.

*Note:* Some round objects, such as lemons, have a preferred floating orientation. This is due to the distribution of the weight within the object. However, this does not affect the location of the buoyant force significantly, so a coin placed just a little off-center on top of the lemon will cause the lemon to rotate until the coin drops off.

### Instructor:

**E.M.Shalika Manoj Ekanayake**  
**Student (Civil Eng.)**  
**College of Engineering, Sri Lanka**  
**E Mail: [shalikamanoj@yahoo.com](mailto:shalikamanoj@yahoo.com)**  
**Web: <http://shalikamanoj.weebly.com/>**