Lesson 5: Deflection in reinforced concrete beams

Content

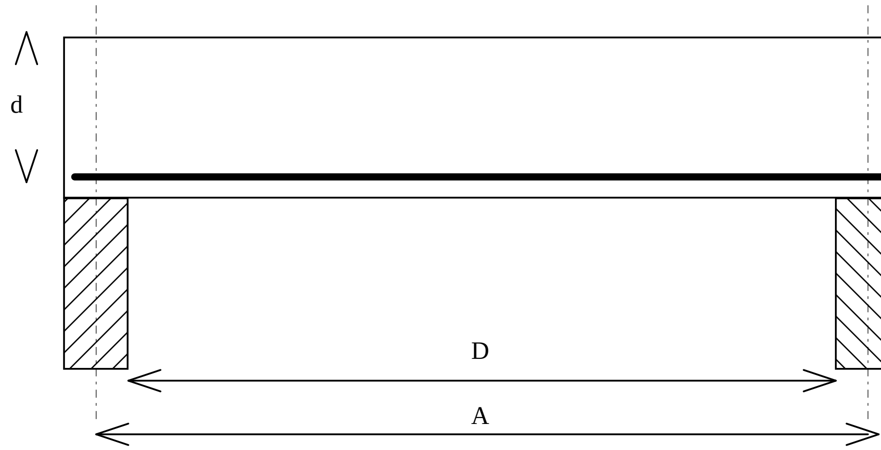
4.1 Introduction to deflection (*Cl. 3.4.6*. BS8110)

As already mentioned we must remember to check **serviceability** as well as **ultimate limit states**. The SLS for deflection considers the performance of the structure under ***Working Loads*** such that the structure, or part of it under consideration, does not deflect excessively causing **unsightly cracking** and **loss of durability**.

BS 8110 details how deflections and the accompanying crack widths may be calculated. But for rectangular beams some simplified procedures may be used to satisfy the requirements without too much effort. This approximate method for rectangular beams is based on permissible ratios of span / effective depth.

4.2 Definitions:

For span read effective span:



Here the effective span is the **LESSER** of:

The distance between centres of bearings, A

OR The CLEAR span plus the effective depth, D+d

BS8110 Deflection Criteria:

1. Total deflection < span / 250

OR 2. For spans up to 10 m, deflection after partitions and finishes < span / 500 or 20 mm, **whichever lesser**.

These criteria are deemed to be satisfied in the following cases:

Table *3.9* BS8110

Support Conditions	Span / Effective depth (rectangular section)	For Initial design
Cantilever	7	6
Simply Supported	20	12
Continuous	26	15

The above **BASIC** ratios apply to spans up to 10 m. For **greater spans** these values should be multiplied by 10 / span. e.g. for 15m span 10 / 15 = 0.66666

These BASIC ratios may be enhanced by provision AND over provision of both tension and compression reinforcement as shown by the following tables.

4.2.1 Tension

Table *3.10* BS8110

Service stress	f_s	M / bd²								
		0.5	0.75	1.00	1.50	2.00	3.00	4.00	5.00	6.00
	100	2	2	2	1.86	1.63	1.36	1.19	1.08	1.01
	150	2	2	1.98	1.69	1.49	1.25	1.11	1.01	.94
250	156	2	2	1.96	1.66	1.47	1.24	1.10	1.00	.94
	200	2	1.95	1.76	1.51	1.35	1.14	1.02	.94	.88
	250	1.9	1.7	1.55	1.34	1.20	1.04	.94	.87	.82
460	288	1.68	1.5	1.38	1.21	1.09	0.95	.87	.82	.78
	300	1.6	1.44	1.33	1.16	1.06	0.93	.85	.8	.76

The values in the above table are derived from:

$$\text{Modification factor } = 0.55 + \frac{(477 - f_s)}{120(0.9 + M/bd^2)} \leq 2.0$$

where:

M = the design ultimate moment

f_s = design service stress of tension reinforcement

$$= \frac{5 f_y A_{s,req}}{8 A_{s,prov}} \times \frac{1}{\beta_b}$$

and β_b = the percentage of moment redistribution

4.2.3 Compression

Table *3.11* BS8110

$100A_{s,prov}/bd$	Factor
0.0	1
.15	1.05
.25	1.08
.35	1.1
.5	1.14
.75	1.2
1.0	1.25
1.5	1.33
2.0	1.4
2.5	1.45
>=3.	1.5

Note: deflection is usually more critical in **slab design** ..

4.3 Initial sizing

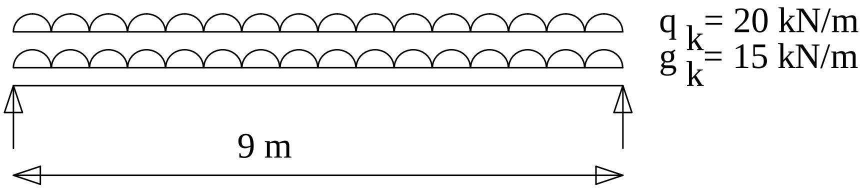
All designers develop their own techniques to help them start to tackle the problem in hand. The following example shows a simple method of **sizing a beam using:**

1. Span/Effective depth ratio to estimate a suitable depth for the section.
2. Limiting the design shear stress ($v=V/bd$) to say 1.2N/mm^2.

As the starting point.

4.3.1 Worked example

A simply supported beam has an effective span of 9 m and supports loads as shown. Determine suitable dimensions for the effective depth and width of the beam.



From the table of Span / d for initial sizing

$$\frac{Span}{d} = 12$$

$$d = \frac{Span}{12} = \frac{9000}{12} = 750mm$$

Total Ultimate Load

$$(1.4g_k + 1.6q_k)span = (1.4 \times 15 + 1.6 \times 20)9 = 477kN$$

Design Shear Force V

$$V = \frac{477}{2} = 238.5kN$$

Limiting *v* to 1.2 N/mm2. Gives *b*

$$v = 1.2 = \frac{V}{bd} = \frac{238.5 \times 10^3}{b \times 750}$$

$$b = \frac{238.5 \times 10^3}{1.2 \times 750} \approx 275mm \text{ (to nearest 25 mm)}$$

Hence a beam of width 275 mm and effective depth 750 mm would be suitable to support the given design loads.

4.4 Reinforcement Details

The code (BS8110) requires the final design to pay attention to:

1. Min and Max reinforcement as a percentage of the gross CSA. (Cl. 3.12.5.3. & 3.12.6.1) - This will on the one hand, help the control of cracking, increase durability, while on the other assist placing and compaction.

$$0.24\% bh \le A_s \le 4\% \text{ when } f_y = 250 N/mm^2$$

$$0.13\% bh \le A_s \le 4\% \text{ when } f_y = 460 N/mm^2$$

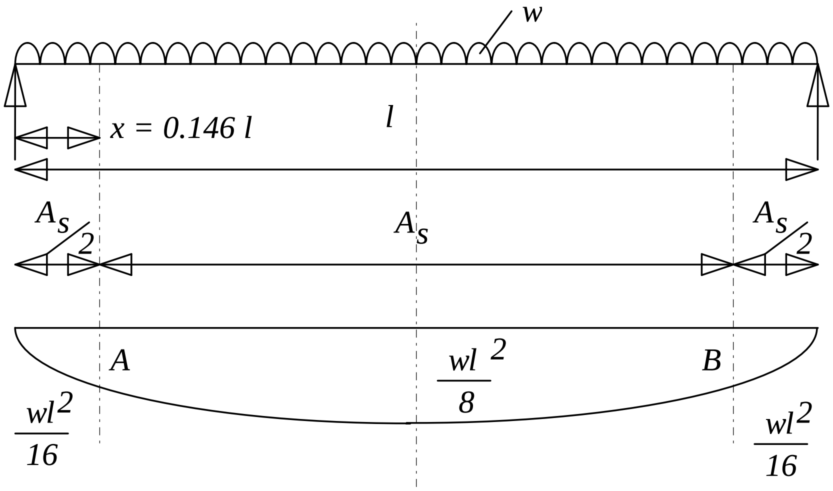
2. Min and Max spacing of tension reinforcement (Cl. 3.12.11.1.) - To allow good compaction and to ensure crack widths do not exceed 0.3 mm respectively.

$$h_{agg} + 5 \text{ mm or bar size} \le s_b \le 300 \text{ mm when } f_y = 250 N/mm^2$$

$$h_{agg} + 5 \text{ mm or bar size} \le s_b \le 160 \text{ mm when } f_y = 460 N/mm^2$$

3. Curtailment and (Cl. 3.12.9.) - To allow for the variation in bending moment decrease towards the supports in a simply supported beam.

THEORETICALLY - 50% of the bars may be curtailed at points A & B as shown below.



PRACTICALLY

However in order to develop any stress in these bars they must be anchored into the concrete. Except at the end supports the bars are therefore normally extended beyond this theoretical cut-off point by:

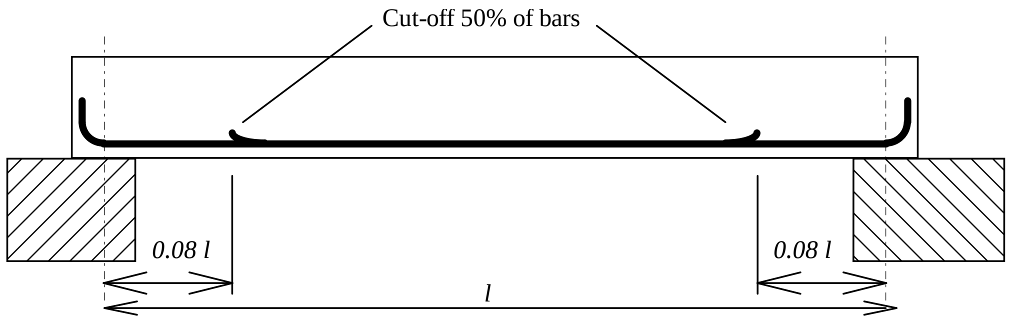
The greater of *'d'* OR *12ϕ*

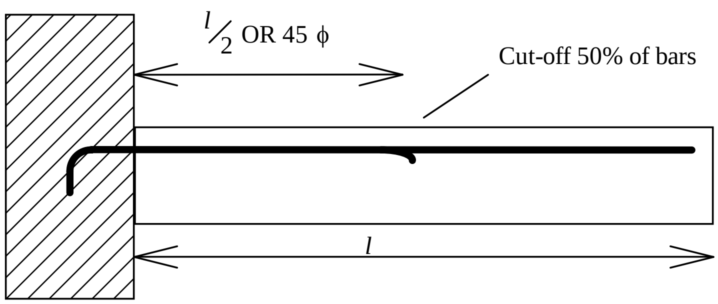
Also where a bar is stopped in the tension zone FULL anchorage length is stipulated:

Table 3.29 - FULL Anchorage lengths as bar size multiples

Reinforcement Type		L_A 25	30	35	>=40
Plain	Tension	39	36	33	31
	Compression	32	29	27	25
Deformed (1)	Tension	51	46	43	40
	Compression	41	37	34	32
Deformed (2)	Tension	41	37	34	32
	Compression	32	29	27	26

Simplified Rules for SS Beams and Cantilevers (Cl. 3.12.10.2.)



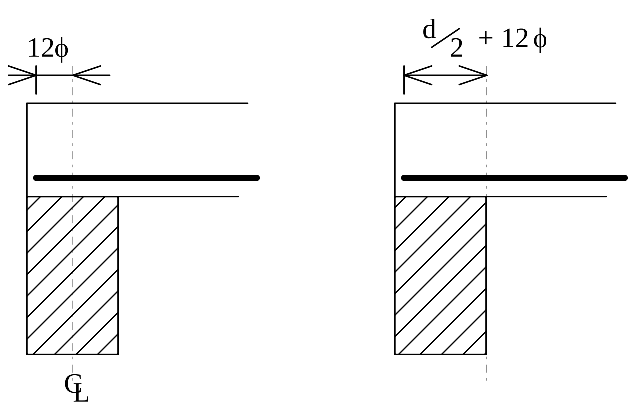


4.5 Anchorage at Supports

Each tension bar will be deemed properly anchored if they extend:

a) 12 ϕ beyond C-L of support

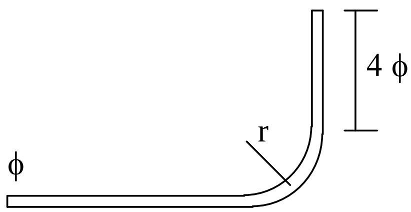
b) 12 ϕ + $d/2$ from face of support

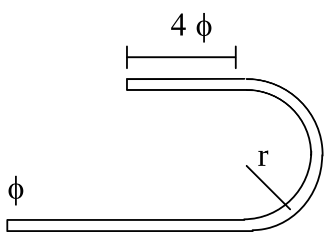


Hooks and bends may be used where necessary to provide adequate anchorage lengths but they must not begin before the centre of support when used to meet condition a) OR before $d/2$ from the face for condition b).

For Mild Steel $r_{min} = 2\phi$

For High Yield Steel $r_{min} = 3\phi$ OR 4ϕ for >=25 mm.





For 90 bend $L_A = 4r < 12$□

For Hook $L_A = 8r < 24$□

4.6 Laps In Reinforcement

To enable continuity of reinforcement at construction joints and to enable bar lengths to be man-handled lapped bars are normally detailed.

Min lap length > 15ϕ OR 300 mm

Tension Laps Use Anchorage Length $L=L_A$. □

where ϕ is the diameter of the smaller bar.

Compression Laps Use 1.25 times the compression anchorage length.